B. Stat (Hons.) & B. Math. (Hons.) Admission Test : 2020

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Short-Answer Type Test

Time: 2 hours

1. Let i be a root of the equation $x^2+1=0$ and let ω be a root of the equation $x^2+x+1=0$. Construct a polynomial
$$f(x) = a_0 + a_1 x + \cdots + a_n x^n$$
where $a_0, a_1, \ldots, a_n$ are all integers such that $f(i+\omega)=0$.

2. Let a be a fixed real number. Consider the equation
$$(x+2)^2(x+7)^2 + a = 0,\ x \in \mathbb{R}$$
where $\mathbb{R}$ is the set of real numbers. For what values of a, will the equation have exactly one double root?

3. Let A and B be variable points on x-axis and y-axis respectively such that the line segment AB is in the first quadrant and of a fixed length $2d$. Let C be the mid-point of AB and P be a point such that

   (a) P and the origin are on the opposite sides of AB and,

   (b) PC is a line segment of length d which is perpendicular to AB.

   Find the locus of P.

4. Let a real-valued sequence $\{x_n\}_{n\geq 1}$ be such that
$$\lim_{n\to\infty} n x_n = 0$$
Find all possible real values of t such that $\lim_{n\to\infty} x_n (\log n)^t = 0$.

5. Prove that the largest pentagon (in terms of area) that can be inscribed in a circle of radius 1 is regular (i.e., has equal sides).

6. Prove that the family of curves
$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$$
satisfies
$$\frac{dy}{dx}(a^2-b^2) = (x + y\frac{dy}{dx})(x\frac{dy}{dx} - y).$$

7. Consider a right-angled triangle with integer-valued sides $a < b < c$ where a, b, c are pairwise co-prime. Let $d = c - b$. Suppose d divides a.
Then

   (a) Prove that $d \leq 2$

   (b) Find all such triangles (i.e., all possible triplets a,b,c) with perimeter less than 100.

8. A finite sequence of numbers $(a_1, \ldots, a_n)$ is said to be *alternating* if

$$a_1 > a_2, \;\; a_2 < a_3, \;\; a_3 > a_4, \;\; a_4 < a_5, \ldots$$

$$\text{or } a_1 < a_2, \;\; a_2 > a_3, \;\; a_3 < a_4, \;\; a_4 > a_5, \ldots$$

   How many alternating sequences of length 5, with distinct numbers $a_1, a_2, \ldots, a_5$ can be formed such that $a_i \in \{1, 2, \ldots, 20\}$ for $i = 1, \ldots, 5$?