ISI M.MATH Admission Test: 2020
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Multiple-Choice Test

Time: 2 hours

Notation.
$\mathbb{R}$ denotes the set of all real numbers.
$\mathbb{C}$ denotes the set of all complex numbers.

1. Let V be a finite dimensional vector space and let W be a proper subspace of V. Let W' be another subspace of V such that $V = W \oplus W'$, i.e. $V = \text{span }(W \cup W')$ and $W \cap W' = \{0\}$. Let $T : V \to V$ be an invertible linear map such that $T(W) \subset W$. Which of the following statements is necessarily true ?

   (a) $T(W') \subset W'$;

   (b) $W' \subset T(W')$;

   (c) $T(W') \cap W = \{0\}$;

   (d) $W' \subset \ker(T)$.

2. Let V be a finite dimensional real vector space, and let $T : V \to V$ be a linear map such that $\text{Range}(T) = \ker(T)$. Which of the following statements in not necessarily true ?

   (a) $T = 0$;

   (b) $T^2 = 0$;

   (c) 0 is an eigenvalue of T;

   (d) All eigenvalues of T are equal to 0.

3. Consider the vector space $\mathbb{R}^n$ equipped with the Euclidean metric d define by

$$d(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^1 /2$$

   Let W be a proper subspace of $\mathbb{R}^n$. Which of the following statements is necessarily true ?

   (a) W is closed.

   (b) W is open.

   (c) W is not closed.

(d) W is neither closed nor open.

4. Let A be a 5×5 real matrix. If $A = (a_{ij})$, let A_ij denote the cofactor of the entry a_{ij}, for $1 \leq i, j \leq 5$. Let A denote the matrix whose (i, j)-th entry is $A_{ij}, 1 \leq i, j \leq 5$. Suppose the rank of A is 3. What is the rank of A?

   (a) 1;
   (b) 3;
   (c) 5;
   (d) 0.

5. For $n \geq 2$, the determinant of the $n \times n$ permutation matrix

   (a) $(-1)^n$;
   (b) $(-1)^{n(n-1)/2}$;
   (c) -1;
   (d) 1.

6. Let $M_2(\mathbb{R})$ denote the vector space of all 2×2 matrices over the field of real number i.e.

$$M_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

   Let $S \subset M_2(\mathbb{R})$ be the subspace defined by

$$S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{R}) : a + c = 0 \right\}.$$

   Then the dimension of S is

   (a) 1;
   (b) 2;
   (c) 3;
   (d) 4.

7. Let V be a finite dimensional real vector space of dimension $n > 1$ and let $W \subset V$ be a subspace of dimension $n - 1$. A linear map from V to $\mathbb{R}$ is called a linear functional on V. Which of the following statements is necessarily true ?

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(a) There does not exist any linear functional on V such that W is the kernel of that linear functional;

(b) W is the kernel of unique linear functional on V;

(c) W is the kernel of a linear functional on V;

(d) There exist a non-zero linear functional on V whose kernel strictly contains W.

8. Let $< \cdots >_n$ denote the standard inner product on the vector space $\mathbb{R}^n$,i.e.

$$< x, y >_n = \sum_{i=1}^{n} x, y_i$$

for vectors $x, y \in \mathbb{R}^n$. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map such that

$$< Tx, Ty >_m = < x, y >_n$$

for all $x, y \in \mathbb{R}^n$. which of the following statements is necessarily true ?

(a) $n \geq m$.

(b) $n \leq m$.

(c) $n = m$.

(d) The map T is onto

9. Let V be a finite dimensional real inner product space and let $T : V \to V$ be a linear map such that $< TxTy > = < x, y >$ for all $x, y \to V$. Suppose $W \subset V$ is a proper subspace of V such that $T(W) \subset W$ Define a subspace $W^\perp$ of V by

$$W^\perp := \{v \in V | < v, w > = 0 \text{ for all } w \in W\}$$

(a) $T(W^\perp)$ is not contained in $W^\perp$.

(b) $T(W^\perp)$ is contained in $W^\perp$.

(c) $T(W^\perp) \cap W^\perp = \{0\}$.

(d) $W^\perp$ is contained in $T(W^\perp)$

10. Let R be a ring with unit such that $a^2 = a$ for all $a \in \mathbb{R}$. Which of the following statements is not neccessarily true ?

(a) $ab = -ba$ for all $a, b, \in \mathbb{R}$,

(b) $a = -a$ for all $a \in \mathbb{R}$,

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(c) R is commutative,

(d) $R = \{0, 1\}$.

11. Let S be a nonempty set, and let $p(S)$ be the power set of S, i.e. $p(S) = \{A | A \subset S|\}$. Define a binary operation $\angle$ on $P(S)$ by $A \angle B := (A \cup B) \setminus (A \cap B)$ for $A, B \in P(S)$. Which of the following statements is necessarily true ?

(a) $(P(S), \triangle)$ is not a group as $\triangle$ is not associative.

(b) $(P(S), \triangle)$ is not a group as there is no identity.

(c) $(P(S), \triangle)$ is an abelian group.

(d) $(P(S), \triangle)$ is a non-abelian group.

12. Let I_1, I_2 be ideals of a commutative ring $\mathbb{R}$. Define the set

$$I_1 + I_2 := \{a + b | a \in I_1, b \in I_2\}.$$

Which of the following statements in not neccessarily true ?

(a) $I_1 + I_2$ is an ideal of $\mathbb{R}$.

(b) $I_1 \subset I_1 + I_2$.

(c) $|I_1 + I_2| = |I_1| + |I_2|$ if $\mathbb{R}$ is finite.

(d) $|I_1 + I_2| = |I_1| \cdot |I_2|$ if $I_1 \cap I_2 = \{0\}$ and $\mathbb{R}$ is finite.

13. Let I be an ideal of a commutative ring $\mathbb{R}$. Define the set

$$\sqrt{I} := \{a \in \mathbb{R} | \text{ There exists } n \geq 1 \text{ such that } a^n \in I\}.$$

Which of the following statements is neccessarily true ?

(a) $\sqrt{I}$ is an ideal.

(b) $\sqrt{I}$ is not an ideal.

(c) $\sqrt{I} = I$.

(d) $\sqrt{I} \subset I$.

14. How many non-isomorphic group are there of order 15?

(a) 1;

(b) 2;

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(c) 3;

(d) 5.

15. Suppose G is a group and $a, b \in G$. which of the following statements is neccessarily true ?

(a) There exist a positive integer n such that $a^n = b^n$;

(b) $(ab)^{-1} = a^{-1}b^{-1}$;

(c) $o(ab) = o(ba)$.

(d) None of the above statements is necessarily true.

16. Let H_1, H_2 be distinct subgroups of a finite abelian group G. Define the subgroup H_1H_2 by $H_1H_2 = h_1h_2|h_1 \in H_1, h_2 \in H_2$. Which of the following statements is neccessarily true ?

(a) $|G| \leq |H_1| + |H_2|$;

(b) $|G \setminus (H_1 \cap H_2)| = |GH_1| \cdot |G \setminus H_2|$,

(c) $|G \setminus H_1| = |G \setminus H_2| \cdot |H_2 \setminus (H_1 \cup H_2)|$.

(d) $|(H_1H_2) \setminus H_1| = |H_2 \setminus (H_1 \cup H_2)|$.

17. Let $P > 3$ be a prime number and let s_p denote the symmetric group on P symbols. How many p-Sylow subgroup are there in S_p?

(a) 1;

(b) p;

(c) 2;

(d) $(P-2)!$.

18. Let $\mathbb{R}$ be a commutative ring with unit and let

$$\mathbb{N} = \{a \in \mathbb{R} | a^n = 0 for some integer n \geq 0\}.$$

Which of the following statements is necessarily true ?

(a) Any prime of $\mathbb{R}$ contains $\mathbb{N}$,

(b) $\mathbb{N}$ is not an ideal,

(c) $\mathbb{N}$ is a prime ideal,

(d) $\mathbb{N} = \{0\}$.

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19. Let $f : \mathbb{R} \to \mathbb{R}$ be twice continuously differentiable, and suppose $\lim_{x\to\infty} f''(x) = 1$. Which of the following statements is necessarily true ?

(a) $\lim_{x\to\infty} \frac{f(x)}{x^2} = 1$,

(b) $\lim_{x\to\infty} \frac{f(x)}{x^2}$ does not exist,

(c) $\lim_{x\to\infty} \frac{f(x)}{x^2} = 2$,

(d) $\lim_{x\to\infty} \frac{f(x)}{x^2} = 1 \setminus 2$.

20. Let $f_n : [0,1] \to \mathbb{R}$ be define by $f_n(x) = (\cos(\pi x))^{2n}$. Which of the following statements is true ?

(a) The sequence $\{f_n\}$ converges uniformly on $0, 1$,

(b) The sequence $\{f_n\}$ converges pointwise on $[0, 1]$ to a function f such that f has exactly one point of discontinuity,

(c) The sequence $\{f_n\}$ converges pointwise on $[0, 1]$ to a function f such that f has exactly two points of discontinuity,

sequence $\{f_n\}$ does not converge pointwise on $[0, 1]$.

21. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous function such that

$$\int_{-1}^{1} f(x)x^{2n}dx = 0$$

for all $n \geq 0$.Which of the following statements is necessarily false ?

(a) $f_{-1}^{1} f(x)^2 dx = f_{-1}^{1} f(-x)^2 dx$.

(b) $\left(\sup_{x\in|-1,1|} f(x)\right) + \left(\inf_{x\in|-1,1|} f(x)\right) = 0$.

(c) $f(0) \neq 0$.

(d) $f(1 \setminus 2)f(-1 \setminus 2) \leq 0$.

22. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x + 1) = f(x) + 1$ for all $x \in \mathbb{R}$. Which of the following statements is necessarily false ?

(a) $\lim_{x+\infty} \frac{f(x)}{x^{1+e}} = 0$ for all $\in > 0$,

(b) $\lim_{x+\infty} \frac{f(x)}{x}$ does not exist,

(c) $\lim_{x+\infty} \frac{f(x)}{x} = 1$,

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(d) $\lim_{x+\infty} \frac{f(x)}{x^{1+e}} = +\infty$ for all $\in > 0$.

23. Let $\{0,1\}^{\mathbb{N}}$ denote the set of all sequence $\{x_n\}$ such that $x_n \in \{0,1\}$ for all $n \geq$. Define a map $f : \{0,1\}^{\mathbb{N}} \to \mathbb{R}$ by

$$f(\{x_n\}) := \sum_{n=1}^{\infty} \frac{x_n}{2^n}.$$

Which of the following statements is true ?

(a) The map f is one-to-one and onto from $\{0,1\}^{\mathbb{N}}$ to $[0,1]$.

(b) The map f is one-to-one and onto from $\{0,1\}^{\mathbb{N}}$ to $[0,1)$.

(c) The map f is onto from $\{0,1\}^{\mathbb{N}} to [0,1]$ and $|f^{-1}(1 \setminus 2)| = 2$.

The map f is onto from $\{0,1\}^{\mathbb{N}}$ to $[0,1]$ and $|f^{-1}(1)| = 2$.

24. Let $f : [0,\infty) \to \mathbb{R}$ be a monotone increasing functions, and define $f_n : [0,\infty) \to \mathbb{R}$ by

$$f_n(x) = f(x+n), x \in [0,\infty)$$

for all $n \geq 1$. Suppose that for some $x_0 \in [0,\infty)$, the limit $\lim_{n\to\infty} f_n(x_0)$, exists. Which of the following statements is necessarily false ?

(a) The sequence $\{f_n\}$ converges pointwise on $[0,\infty)$.

(b) The sequence $\{f_n\}$ converges uniformly on $[0,\infty)$.

(c) The limit $\lim_{x\to\infty} f(x)$ exists.

(d) The function f is unbounded on $[0,\infty)$.

25. Let X, Y be set and let $f : X \to Y$ be a function. Let $\{S_i\}_{i\in I}$ be a family of subsets of X,i.e. $S_i \subset X$ for all $i \in I$, Where I is an index set. Which of the following statements is not necessarily true ?

(a) $F(\cup_i \in IS_i) \subset \cup_{i\in I} f(S_i)$.

(b) $F(\cup_i \in IS_i) \supset \cap_{i\in I} f(S_i)$.

(c) $F(\cap_i \in IS_i) \subset \cap_{i\in I} f(S_i)$.

(d) $F(\cup_i \in IS_i) \supset \cup_{i\in I} f(S_i)$.

26. Let S be the set of all those nonnegative real numbers α with the following property: if $\{x_n\}$ is a sequence of nonnegative real number such that $\sum_{n=1}^{infty} x_n < +\infty$, then we also have $\sum_{n=1}^{\infty} \frac{\sqrt{x_n}}{n^\alpha} < +\infty$. Which of the following statements is true ?

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(a) $S = \emptyset$.

(b) $S \supset (1/4), \infty$.

(c) $S \supset (1/2, \infty)$.

(d) $S \subset (3/4, \infty)$.

27. Let X be a finite set. Let $P(X)$ be the power set of X, i.e. the set whose elements are all subset of X. Which of the following defines a metric on the power set $P(X)$?

(a) $d(V, W) = |(V \cup W) \setminus (V \cap W)|$.

(b) $d(V, W) = |V \cap W|$.

(c) $d(V, W) = |V \setminus W|$.

(d) $d(V, W) = |V \cup W|$.

28. The tangent line to the curve $2_x^6 + y^4 = 9_{xy}$ at the point $(1, 2)$ has slope

(a) 3/23,

(b) 6/23,

(c) 9/23,

(d) 4/7.

29. Consider the following statements:

(a) If $\sum_n a_n$ and $\sum_n b_n$ are convergent, then $\sum_n a_n b_n$ is convergent.

(b) If $\sum_n a_n$ is convergent and $\sum_n b_n$ is absolutely convergent, then $\sum_n a_n b_n$ is absolutely convergent.

(c) If $a_n \geq 0$ for all n, $\sum_n a_n$ is convergent, and $\{b_n\}$ is bounded sequence, then $\sum_n a_n b_n$ is absolutely convergent, then $\sum_n a_n b_n$ is absolutely convergent. Which of the following statements is true ?

(a) All of the statements are true,

(b) Statement is true but statement is false,

(c) Only statements and are true,

(d) Only statements and are true.

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30. Consider the metric space $(\mathbb{N} = \mathbb{N} \cup \{\infty\}, d)$, Where the metric d is define by $d(m,n) = |\frac{1}{m} - \frac{1}{n}|$ for $m, n \in \mathbb{N}$, and $d(n, \infty) = 1/n$ for $n \in \mathbb{N}$. Let $f : \mathbb{N} \to \mathbb{R}$ be a continuous function between metric space (Where $\mathbb{R}$ is equipped with its usual metric). Which of the following statements is necessarily false ?

(a) The metric space $\mathbb{N}$ is compact,

(b) The function f is unbounded,

(c) The function f is uniformly continuous,

(d) For any $x \in \mathbb{R}$, the set $f^{-1}(\{x\})$ is compact.

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